Explanatory remarks concerning oblique shock fronts on A. Busemann's paper, "Die achsensymmetrische kegelige ueberschallströmung." Luftfahrtforschung, Vol. 19, p. 137.

Oblique shock results when a flow is deflected by a "wedge".

The shock is assumed to be steady, the velocity in front of it is $u_1 = q_1$, $v_1 = o$; the velocity past the shock $u_2 = q_2 \cos\beta$, $v_2 = q_2 \sin\beta$. To formulate the transition relations one must first determine the "critical speed".

$$a_* = \sqrt{\frac{2}{\kappa + 1} a_1^2 + \frac{\kappa - 1}{\kappa + 1} q_1^2}$$

(the exponent γ is called κ in Busemann's article). The quantity $a_1 = \sqrt{\kappa p_1/\rho_1}$ is the sound velocity of air with the pressure p_1 and the density (i.e. mass p. u. volume) ρ.

From relation (17) in Busemann's paper one could find analytically v_2 ~~when~~ if u_2 and u_1 were given. If, however,

u_1 and the angle β of the new direction are given one must resort to reading off from diagram 7 in the paper.

In diagram 7, u and v are the coordinates; to fix the unit, the circle $q = a_*$ is indicated. The point (u_1, o) is then to be located on the axis. Two "shock polars" through such points (u_1, o) are shown in the figure. The lower one refers to the simple oblique shock now considered. On this curve one should find the intersection with the line through the origin under the desired angle β. If ~~[illegible]~~ β is sufficiently small, there are two intersections; the one with the greater speed q is generally assumed to occur. Once thus u_2, v_2 and hence q_2 are found, the pressure can be calculated from

$$\frac{p_2}{p_1} = 1 + \kappa \frac{u_1(u_1-u_2)}{a_1^2}$$

The density, i. e. mass p. u. volume, of the air in state (2) could be calculated from

$$\frac{\rho_2}{\rho_1} = \frac{u_1(u_1-u_2)}{u_2(u_1-u_2)-v_2^2}$$

or

$$\frac{\rho_2}{\rho_1} \qquad \frac{(\quad +1)p_2+(\quad -1)p_1}{(\quad -1)p_2+(\quad +1)p_1}$$

which may serve as a check.

Busemann has apparently a different procedure in mind to determine the pressure pass the shock: One is first to calculate the pressure that would result if the velocity (u_2, v_2) were obtained isentropically and then multiply by a reduction factor labeled as "p_o'/p_o" in his diagrams. Accordingly one has to calculate

$$\left(\frac{q_2}{a_*}\right)^2 = \left(\frac{u_2}{a_*}\right)^2 + \left(\frac{v_2}{a_*}\right)^2$$

$$\left(\frac{a_2}{a_*}\right)^2 = \frac{\kappa+1}{2} - \frac{\kappa-1}{2}\left(\frac{q_2}{a_*}\right)^2$$

$$\frac{p_2}{p_1} = \frac{p_o'}{p_o}\left(\frac{a_2^2}{a_1^2}\right)^{\frac{\kappa}{\kappa-1}}$$

The shock direction is perpendicular to the line joining the points (u_1, o) and (u_2, v_2) and can thus be read from the diagram. Or, one calculates the angle γ from

$$\tan\gamma = \frac{u_1-u_2}{v_2}$$

Example. Suppose the incoming flow has the Mach number $q_1/a_1 = 1.91$. Then one finds $q_1/a_* = 1.59$. If now the angle $\beta = 9^\circ$ is desired, one reads off from the figure the point of intersection $u_2/a_* = 1.[illegible]0$, $v_2/a_* = .22$. Using the formulas above one obtains $p_2/p_1 = 1.6$, $\rho_2/\rho_1 =$ [illegible]. Further one finds $\gamma = 40^\circ$.

If the flow is deflected by a cone, a conical shock front will occur. The relations between velocities

pressures and densities at both sides of such a conical shock are the same as for two dimensional shocks, but the flow immediately after having passed the conical shock has not yet acquired the direction of the conical obstacle, cf. fig. 6. The velocity (u_3, v_3) at this cone can be read off from the upper diagram in figure 7 by intersecting the line

through the origin under the angle β with the "shock curve" starting at the point (u_1, o). Thus u_3/a_* and v_3/a_* are obtained; then $(q_3/a_*)^2 = (u_3/a_*)^2$ and $(a_3/a_*)^2 = \frac{\kappa+1}{2} - \frac{\kappa-1}{2}(q_3/a_*)^2$ can be computed. To determine the pressure at the cone one uses the reduction factor labeled as p_o'/p_o in the diagram and finds

$$p_3/p_1 = (p_o'/p_o)\,(a_3^2/a_1^2)^{\kappa/\kappa-1}$$

To check this result one could proceed as follows: One determines the velocity $(u_2/a_*,\ v_2/a_*)$ past the shock as for the wedge shock and determines p_2/p_1 and ρ_2/ρ_1 as before. From $p_3/\rho_3^{\kappa} = p_2/\rho_2^{\kappa}$ and $\kappa p_3/\rho_3 = a_3^2$, one derives the relation

$$p_3/p_1 = (a_3/a_1)^{2\kappa/\kappa-1}\,(\rho_2/\rho_1)^{\kappa/\kappa-1} \Big/ (p_2/p_1)^{1/\kappa-1}\ .$$

The angle of shock direction σ can be read off from figure 8.

As an example, it may again be assumed that the air stream ahead of the cone has the Mach number $q_1/a_1 = 1.91$ hence $q_2/a_* = 1.59$. Assuming a half angle of the cone $= 21^\circ$ one finds

$$u_3/a_* = 1.28, \quad v_3/a_* = .51$$

Then one finds $p_3/p_1 = 1.8$ and the angle of the shock direction then is as before $\delta = 40^\circ$.

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